Optimization Semester Report

MAT2407

Tran Dang Quang Minh, K68A2 Mathematics and Computer Science

Vietnam National University
Hanoi University of Science
Faculty of Mathematics, Mechanics and Informatics

Hanoi, December 2025

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Optimal Path-finding Model under time-dependent rainfall and local flooding conditions considering the continuous movement of convective clouds.

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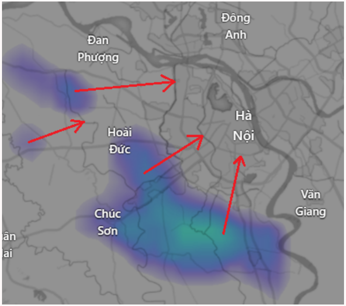

Figure 1.1: *Convective clouds illustrated on windy.com*

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Inspiration

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Heavy flooding in Hanoi on October 1st, 2025.

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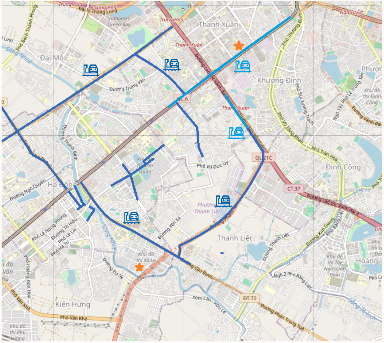

Figure 1.2: *Flooded (light blue) and Heavily flooded (blue) streets on October 1st, 2025*

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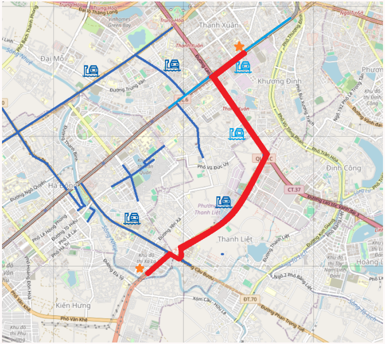

Figure 1.3: *Normal optimal route from HUS to Xala Urban Area (red line)*

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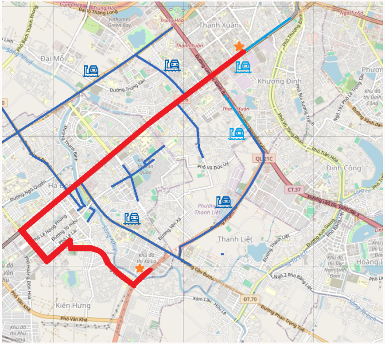

Figure 1.4: *Optimal route from HUS to Xala Urban Area in flooding conditions (red line)*

Report Topic

Simplifying Given Problem Statements

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Convert the map to a Flat Undirected Graph with vertices set V and edges set E.

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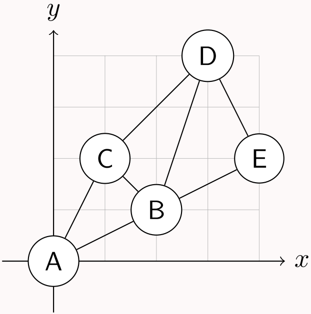

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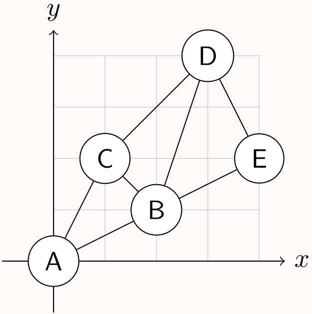

Each vertex i is located at (x_i, y_i).

Each edge e_{ij}, which connects vertices $i(x_i, y_i)$ and $j(x_j, y_j)$, has length:

$$l_{ij} = l_{ji} = 1000 \cdot \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

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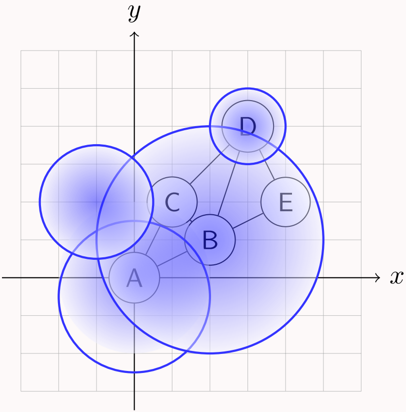
Considering the i-th convective cloud as a circle $B_{r_i}(x_i, y_i)$.



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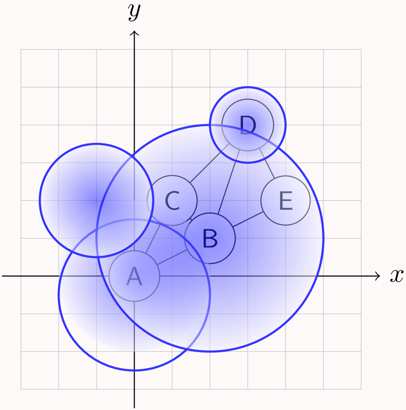
Simplifying Given Problem Statements

The i-th convective cloud has a rainfall rate of $R_i(mm/h)$ at every point.



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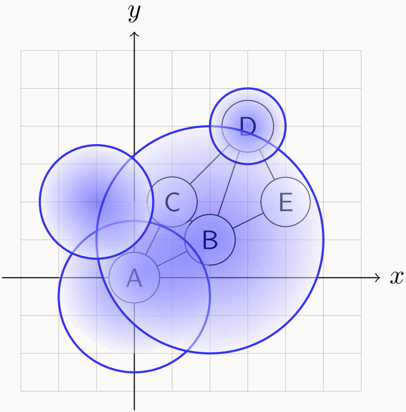
The i-th cloud continuously moving towards vector $v_i = (\Delta_x, \Delta_y)$ every second.



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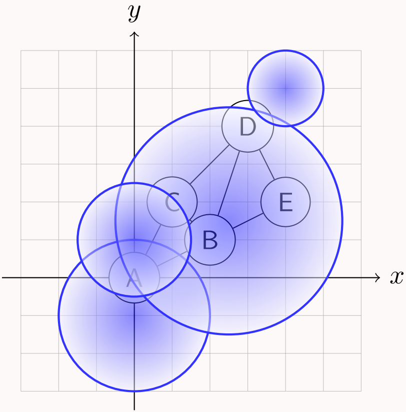

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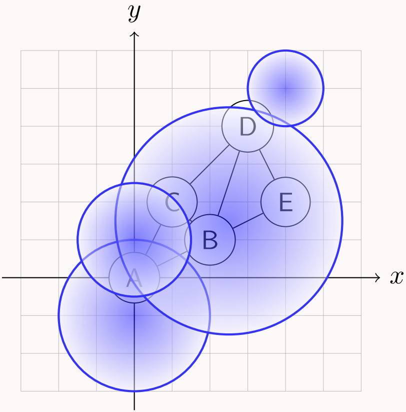
Let $K_{ij}(t_0)$ be the set of clouds that cross the edge e_{ij} at time t_0.



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Consider the rainfall rate on edge e_{ij} at time t_0 defined as

$$r_{ij}(t_0) = \begin{cases} \max\limits_{k \in K_{ij}(t_0)} R_k, & \text{if } K_{ij}(t_0) \neq \varnothing, \\ 0, & \text{if } K_{ij}(t_0) = \varnothing. \end{cases}$$

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Now we have the full problem's input...

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- Each convective cloud $B_{r_i}(x_i, y_i)$ has properties:
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  + $(\Delta_{x_i}, \Delta_{y_i})$: The movement vector of the cloud every second (in meters).

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  + (x_i, y_i): The center of the cloud,
  + $(\Delta_{x_i}, \Delta_{y_i})$: The movement vector of the cloud every second (in meters),
  + r_i: The radius of the cloud (m),
  + R_i: The overall rainfall rate of the cloud (mm/h).

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Simplifying Given Problem Statements

- The problem can now be stated as:

Find the optimal (in fuel usage) route from a vertex (x_i, y_i) *to another vertex* (x_j, y_j) *avoiding any streets that flooded higher than* $H_{max}(cm)$ *such that the overall travel time does not exceed* $T_{max}(h)$*. Suppose that the cost of fuel per kilometer is* $C = 0.75\ \$/km$*.*

Preprocessing Inputs

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Data Processing

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- The time cost of the street e_{ij} is:

$$t_{ij} = \frac{1}{3600} \cdot \frac{l_{ij}}{v_{ij}} \ (h)$$

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$$V_{ij}(t_0) = 0.001 \cdot r_{ij}(t_0) \cdot A_{ij} \ (m^3/h)$$

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- Then, the flood height of the street e_{ij} at time t_1 after $\Delta t = t_1 - t_0(h)$ is:

$$H_{ij}(t_1) = \begin{cases} 0, & t_1 = 0, \\ H_{ij}(t_0) + \Delta t \cdot \frac{V_{ij}(t_0) - d_{ij}}{A_{ij}}, & V_{ij}(t_0) > d_{ij}, \ \ (m) \\ \max(0, H_{ij}(t_0) - \Delta t \cdot \frac{d_{ij} - V_{ij}(t_0)}{A_{ij}}), & V_{ij}(t_0) \le d_{ij}. \end{cases}$$

Modeling Process

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Setting up Variables

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For each street e_{ij}, denotes a binary variables x_{ijk}:

$$x_{ijk} = \begin{cases} 1, & \text{the street } e_{ij} \text{ is in the optimal route and we go through it at } t = k, \\ 0, & \text{otherwise.} \end{cases}$$

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$$y_{ik} = \begin{cases} 1, & \text{the traveler is at the } i\text{-th node at } t = k, \\ 0, & \text{otherwise.} \end{cases}$$

Denotes a discrete time variable $k \in T$, T is a dimension of time from 0 to T_{max}. Consider E_{ans} the set of streets in the optimal route. If there doesn't exist a valid route, returns an empty set. Otherwise, returns the set of streets from the start point to the destination point.

Modeling Process

Objective Function

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The objective is to determine the lowest fuel cost, so the Objective Function is:

$$\mathit{min} \quad \sum_{(i,j)\in E, k\in T} 0.001 \cdot x_{ijk} \cdot l_{ij} \cdot C$$

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or to be simpler, $\textit{min} \quad \sum_{(i,j)\in E, k\in T} x_{ijk} \cdot l_{ij}$

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This confirms that there's only a single path from the source to the destination.

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$$\sum_{(i,j)\in E,k\in T} t_{ij} \cdot x_{ijk} \leq T_{max}$$

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3. Flood Constraints:

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3. Flood Constraints:

$$H_{ij}(t_{si}) \leq H_{max} \forall (i,j) \in E,$$

t_{si}: time to travel from source node to the i-th node.

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$$H_{ij}(t_{si}) \leq H_{max} \forall (i,j) \in E,$$

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Since there's a t_{si} variable in the constraint and the traveling time between nodes are different, it would be a nonlinear constraint.

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Hence, we have to linearize the time by dt, such that dt is the smallest duration (in second) we store a state. For example, if $dt = 5$, we store the state of conditions every 5 seconds.

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Since there's a t_{si} variable in the constraint and the traveling time between nodes are different, it would be a nonlinear constraint.

Hence, we have to linearize the time by dt, such that dt is the smallest duration (in second) we store a state. For example, if $dt = 5$, we store the state of conditions every 5 seconds.

Then, if the t_{si} isn't divisible by dt, we choose the maximum t that can be divided by dt.

Modeling Process

Constraints

4. Other Constraints:

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$$y_s^0 = 1, \tag{1}$$

$$y_i^0 = 0, \quad \forall i \in V \setminus \{s\}. \tag{2}$$

$$x_{ijk} \le y_i^k, \quad \forall (i,j) \in E,\ \forall k \in T, \tag{3}$$

$$y_j^{k+t_{ij}} \ge x_{ijk}, \quad \forall (i,j) \in E,\ \forall k \text{ such that } k + t_{ij} \le K. \tag{4}$$

$$\sum_{i \in V} y_i^k \le 1, \quad \forall k = 0, 1, \ldots, K. \tag{5}$$

$$\sum_{k=0}^{K} y_i^k \le 1, \quad \forall i \in V. K = \left\lfloor \frac{T_{\max} \cdot 3600}{dt} \right\rfloor. \tag{6}$$

Modeling and Displaying

Modeling and Displaying

Modeling and Displaying

We can now use gurobipy to run the model on python, and matplotlib to display the map.

Modeling and Displaying

Modeling and Displaying

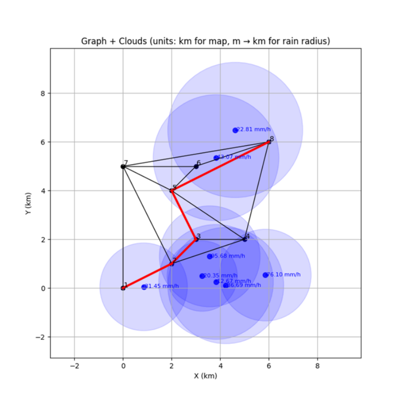

Figure 4.1: *An example of a way to go from the 1st Node to the 8th Node*

Modeling and Displaying

Modeling and Displaying

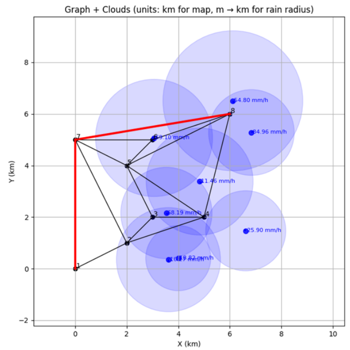

Figure 4.2: *Another way to go from the 1st Node to the 8th Node in a different scenario*

Modeling and Displaying

Modeling and Displaying

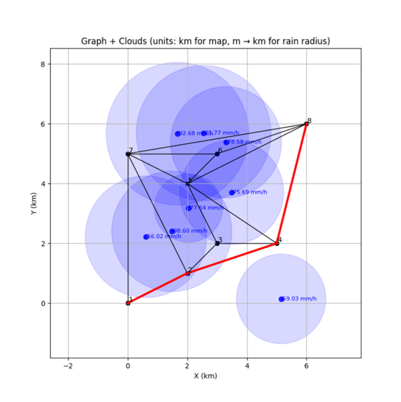

Figure 4.3: *Another way to go from the 1st Node to the 8th Node in a different scenario*

Conclusion

Conclusion

Conclusion

- This work formulates the route planning problem under rainfall-induced flooding as a **time-expanded Mixed Integer Linear Programming (MILP)** model.
- The proposed model explicitly incorporates **dynamic rainfall, cloud movement, road drainage capacity, and flood accumulation over time**.
- By discretizing time, the originally nonlinear flood constraint is successfully linearized, allowing the problem to be solved using standard MILP solvers such as **Gurobi**.
- The obtained solution guarantees a **single continuous path** from the source to the destination while satisfying both **travel time** and **flood safety constraints**.

Conclusion

Comparison with Other Approaches

- Classical shortest-path algorithms such as **Dijkstra or A*** are efficient for static networks but fail to capture **time-dependent flooding and dynamic constraints**.
- Heuristic or metaheuristic methods (e.g., **Genetic Algorithms, Ant Colony Optimization**) can handle dynamic environments but **do not guarantee optimality**.
- In contrast, the proposed MILP model provides a **globally optimal solution** under well-defined constraints, making it suitable for **risk-sensitive and safety-critical routing**.
- The trade-off lies in computational complexity, as MILP scalability is limited compared to graph-based heuristics.

Conclusion

Efficiency and Future Work

- Numerical experiments show that the MILP-based approach is effective for **small- to medium-scale networks** with fine temporal resolution.
- The model accurately avoids flooded streets by accounting for **rainfall accumulation and drainage dynamics**.
- However, the time-expanded formulation leads to a rapid growth in the number of variables and constraints as the network size or time horizon increases.
- Future work includes **model reduction techniques**, **rolling-horizon optimization**, and hybrid approaches combining **MILP with heuristic routing algorithms**.

Thank you!